TRAVERSE COMPUTATION ON THE UTM PROJECTION FOR SURVEYS OF LIMITED EXTENT

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ABSTRACT
Given a set of cadastral traverse measurements reduced to a local plane, grid coordinates (E, N) can be computed in two ways: (i) reduce the traverse measurements to a set of plane bearings and distances on the Universal Transverse Mercator (UTM) projection plane and then use plane trigonometry or (ii) compute geodetic coordinates (ϕ, λ) directly using the *direct* and *inverse* cases on the ellipsoid and then transform these to grid coordinates. The first method (computation on the UTM plane) generally requires iteration and is slow; the second method (computation on the ellipsoid) is quicker. If the survey area is relatively small, say less the 25 square kilometres, certain approximations may be made that makes the first way a relatively simple exercise that avoids the need to deal with geodetic coordinates. The Intergovernmental Committee on Surveying and Mapping (ICSM) and Geoscience Australia have provided Microsoft® Excel spreadsheets for the calculations and this paper describes the method of computation suitable for cadastral surveys of limited extent.

INTRODUCTION
In Australia, topographic mapping and coordination is based on rectangular coordinate grids (east *E,* north *N*) overlaying conformal projections of latitudes ϕ and longitudes λ of points related to geodetic datums. There are two geodetic datums of interest: the new Geocentric Datum of Australia (GDA) and the old Australian Geodetic Datum (AGD), one conformal map projection: the UTM, and two grids: the new Map Grid Australia (MGA) and the old Australian Map Grid

(AMG). Hence we have the coordinate "pairs" AGD/AMG and GDA/MGA. There have been several "realizations" of geodetic datums in Australia – a realization being the actual determination of coordinates (ϕ, λ) related to a reference ellipsoid, by the mathematical adjustment of measurements between stations in the national geodetic network. The first of these was in 1966 and the second in 1984; both being realizations of the AGD and known as AGD66 and AGD84 with grid coordinates designated AMG66 and AMG84. The AGD is a topocentric datum that has now been superseded by the GDA with a realization designated GDA94 with grid coordinates MGA94. In 1995 the Australian government proclaimed the new datum and produced a geodetic coordinate set designated GDA94 referred to the reference ellipsoid of the Geodetic Reference System 1980 (GRS80) and located with respect to the International Terrestrial Reference Frame 1992 (ITRF92) at the epoch 1994.0.

In Australia, coordinate transformations $(\phi, \lambda \Leftrightarrow E, N)$ as well as calculation of grid convergence γ and point scale factor k are defined by Redfearn's formula (Redfearn 1948). Calculations using these formula can be easily done using Microsoft® Excel spreadsheets available on-line via the Internet at the Geoscience Australia website (http://www.ga.gov.au/) following the links to Geodetic Calculations then Calculate Bearing Distance from Latitude Longitude. At this web page the spreadsheet **Redfearn.xls** is available for use or downloading. Alternatively, the ICSM has produced an on-line publication *Geocentric Datum of Australia Technical Manual* Version 2.2 (GDA Technical Manual, ICSM 2002) with a link to **Redfearn.xls**

Computations on the reference ellipsoid are divided into two cases, (i) the *direct* case: given ϕ, λ of point 1 and the azimuth α and geodesic distance s to point 2, compute ϕ, λ of point 2, and (ii) the *inverse* case: given ϕ, λ of points 1 and 2, compute the azimuth and geodesic distance between them. The direct and inverse cases on the ellipsoid are equivalent to the familiar plane coordinate calculations "radiations" and "joins". Excel spreadsheets for the direct and inverse cases on the ellipsoid are available at the Geoscience Australia website following the links to Geodetic Calculations then Calculate Bearing Distance from Latitude Longitude. At this web page the spreadsheet **Vincenty.xls** is available for use or downloading. Alternatively, the GDA Technical Manual has a link to **Vincenty.xls** This paper is not concerned with computations on the ellipsoid, instead, since the survey area is limited in extent, all computations will be done on the UTM projection plane using a simplified approach.

The GDA Technical Manual is a source of valuable information, references, guidelines, computation formula, and Excel spreadsheets. Also, two recent publications may be useful; one in the *Trans Tasman Surveyor* by Will Featherstone and Jean Rüeger (Featherstone & Rüger 2000) and the other in *The Australian Surveyor* by Will Featherstone and John Kirby (Featherstone & Kirby 2002). These papers describe the reduction of traverse measurements to the ellipsoid and traverse computations on the ellipsoid and UTM plane. In addition to these publications, the present author has provided two documents for distribution:

(i) *Traverse Computation on the Ellipsoid and on the Universal Transverse Mercator projection* (Deakin 2005a) and

(ii) *Traverse Computation: Ellipsoid versus the UTM projection* (Deakin 2005b)

The aforementioned publications, Featherstone & Kirby 2002 and Deakin 2005a, 2005b make it clear that traverse computation on the ellipsoid is a quicker and more direct method than traverse computation on the UTM plane and by way of example use the familiar *Buninyong–Flinders Peak* traverse. This traverse has been used for demonstration of computations in technical manuals published over the years: The Australian Map Grid Technical Manual (NMC 1972), The Australian Geodetic Datum Technical Manual (NMC 1985) and the current manual, The GDA Technical Manual.

These papers and technical manuals are replete with formula and terminology that is often confusing to the cadastral surveyor who is not generally concerned with the intricacies of geodesy, computations on the ellipsoid or traversing long distances. Furthermore, they rarely contain "approximate methods" that may simplify computations for cadastral surveys of limited extent. This paper presents a simplified method of computation of MGA coordinates appropriate for surveys of limited extent, which in the context of this paper is taken to be:

(i) A survey of a property (rural or urban) including connections to coordinated reference marks that would fit within a square of 5 kilometres by 5 kilometres (25 square kilometres),

(ii) Individual traverse lines are less than 1 kilometre in length,

(iii) Traverse distances have been reduced to a local horizontal plane, i.e., the horizontal component of Total Station EDM distances have been recorded,

(iv) Traverse bearings are related in some way to an existing survey (possibly connected to true north or magnetic north) and within 15° of MGA grid bearings,

Most cadastral surveys, rural or urban would fit within these parameters and in addition we will assume our survey area is near the boundary of a UTM zone (where *arc-to-chord* corrections and *point scale factors* are a maximum) and the average elevation of the area is approximately 500 metres above the Australian Height Datum (AHD).

With the above restrictions, this paper will cover the necessary corrections to traverse measurements with explanations of any simplifications made and outline a practical method of computing MGA grid coordinates (E, N) using a calculator and the spreadsheets and software provided by the ICSM and Geoscience Australia.

As a preliminary, a brief description of the UTM projection and some terminology and equations will be given. This can also be found in Deakin (2005a) and the GDA Technical Manual. This will be followed by a discussion of scale factors (point scale factor and combined scale factor) and the reduction of distances to the UTM projection plane. Finally, a worked example of a traverse connecting two known points (PM's with grid coordinates) will be shown.

THE UNIVERSAL TRANSVERSE MERCATOR (UTM) PROJECTION

The *Transverse Mercator* (TM) projection is a conformal projection, i.e., the scale factor at a point is the same in every direction, which means that shape is preserved, although this useful property only applies to infinitesimally small regions of the Earth's surface. Meridians and parallels of the ellipsoid are projected as an orthogonal network of curves, excepting the equator and a central meridian, which are projected as straight lines intersecting at right angles. The intersection of the equator and the central meridian is known as the *true origin* of coordinates and the scale factor along the central meridian is constant.

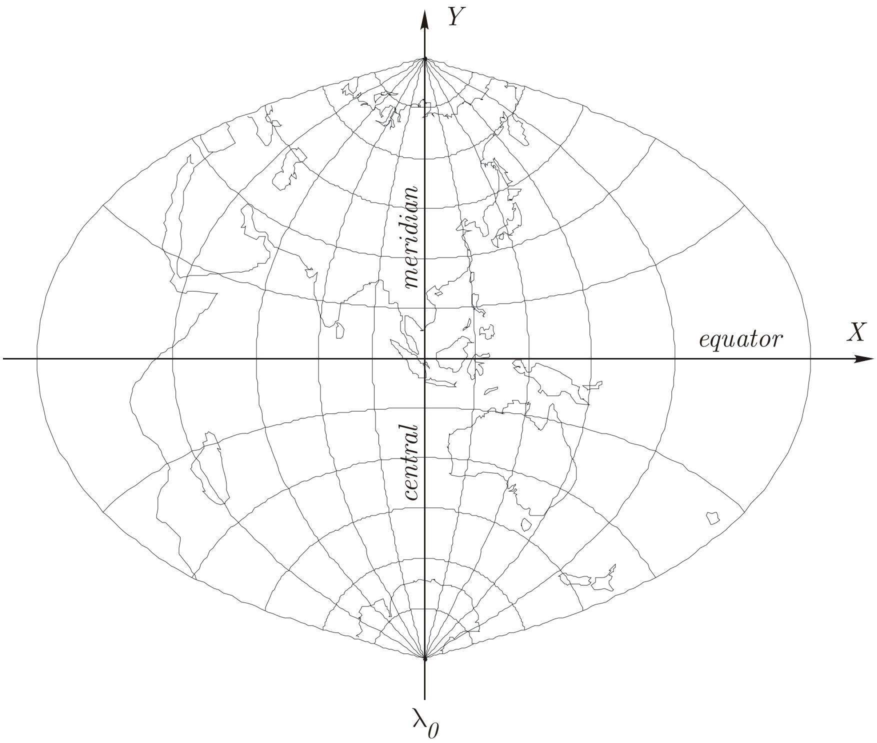


Figure 1. Transverse Mercator projection of part of the ellipsoid.
Central meridian $\lambda_0 = 105^\circ$, graticule interval 15°

The TM projection is very useful for mapping regions of the Earth with large extents of latitude, but for areas away from the central meridian, distortions increase rapidly. To limit the effects of distortion, TM projections are usually restricted to small zones of longitude about a central meridian λ_0. The *Universal Transverse Mercator* (UTM) projection is a TM projection of the ellipsoid with defined zone widths of 6° of longitude (3° either side of the central meridian), a zone numbering system (60 zones of 6° width, with zone 1 having a central meridian 177° W and zone 60 having a central meridian of 177° E), a central meridian scale factor $k_0 = 0.9996$ and a true origin of coordinates for each zone at the intersection of the equator and the central meridian.

To make coordinates positive quantities, each zone has an origin of East and North coordinates (known as the *false origin*) located 500,000 m west along the equator from the true origin for the northern hemisphere, and 500,000 m west and 10,000,000 m south of the true origin for the southern hemisphere.

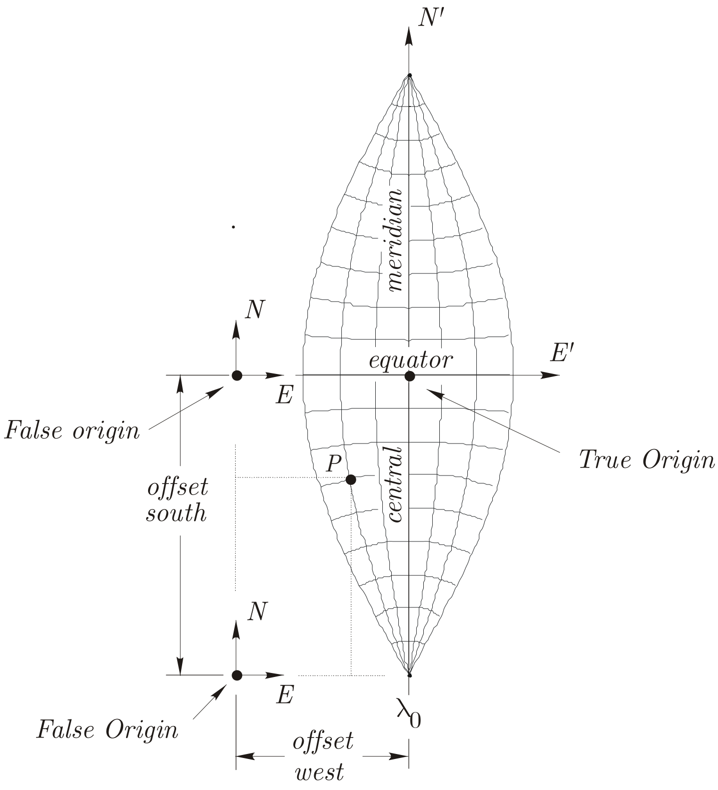


Figure 2 Schematic diagram of a UTM zone showing false origins for the northern and southern hemispheres

Figure 2 shows a schematic diagram of a UTM zone of the Earth. In the southern hemisphere the point P will have negative coordinates E', N' related to the true origin at the intersection of the central meridian and the equator. P has positive E, N coordinates related to the false origin 500,000 m west and 10,000,000 m south of the true origin. True origin and false origin coordinates in the southern hemisphere are related by

$$\begin{aligned} E' &= E - 500,000 \\ N' &= N - 10,000,000 \end{aligned} \tag{1}$$

Figure 3 shows two points P_1 and P_2 on a UTM projection with grid coordinates E_1, N_1 and E_2, N_2. The geodesic s between P_1 and P_2 on the ellipsoid is projected as a curved line concave to the central meridian and shown on the diagram as the *projected geodesic.* The *plane distance* L is the straight line on the projection and

$$L = \sqrt{\left(E_2 - E_1\right)^2 + \left(N_2 - N_1\right)^2} \tag{2}$$

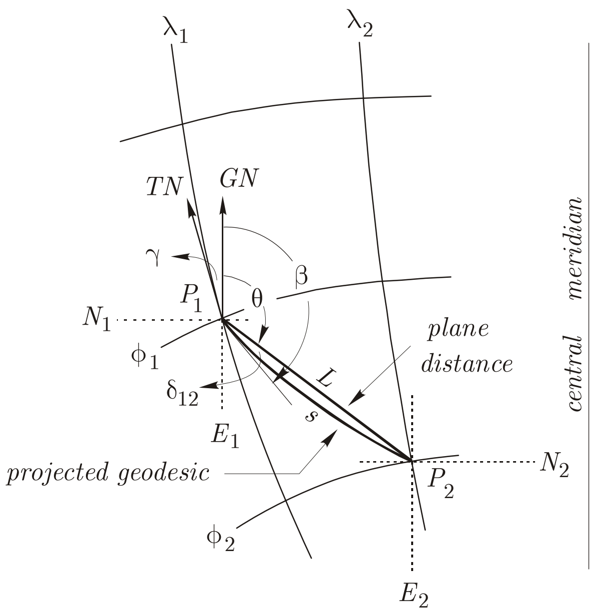


Figure 3. The projected geodesic

The *Line Scale Factor* K is defined as the ratio of plane distance to geodesic distance

$$K = \frac{L}{s} \tag{3}$$

and the Line Scale Factor can be computed from

$$K = k_0 \left[1 + \frac{E_1'^2 + E_1'E_2' + E_2'^2}{6r_m^2} \left\{ 1 + \frac{E_1'^2 + E_1'E_2' + E_2'^2}{36r_m^2} \right\} \right] \tag{4}$$

where $r_m^2 = \rho \nu k_0^2$ and ρ, ν are computed for $\phi_m = (\phi_1 + \phi_2)/2$. Equation (4) is given in various technical manuals (NMC 1972, NMC 1985 and ICSM 2002) and is regarded as accurate to 0.1 ppm over any 100 km line in a UTM zone. Bomford (1962) compared this formula with others over a known test line and recommended its use. For most practical purposes, the term in braces { } in equation (4) is omitted as its effect is negligible. For a line 100 km in length running north and south on a zone boundary the error in neglecting this term is about 0.25 ppm (NMC 1985).

In Figure 3, *Grid North* (*GN*) is parallel to the direction of the central meridian and *True North* (*TN*) is the direction of the meridian. The angle between True North and Grid North is the *grid convergence* γ. The clockwise angle from Grid North to the tangent to the projected geodesic at P_1 is the *grid bearing* β and the *azimuth* α is the clockwise angle from True North to the tangent to the projected geodesic. Grid bearing and Azimuth are related by

$$\beta = \alpha + \gamma \tag{5}$$

By convention, in Australia, the grid convergence is a negative quantity west of the central meridian and a positive quantity east of the central meridian.

In Figure 3, the *plane bearing* θ is the clockwise angle from Grid North to the straight line joining P_1 and P_2. The plane bearing is computed from plane trigonometry as

$$\theta = \tan^{-1} \left(\frac{E_2 - E_1}{N_2 - N_1} \right) \tag{6}$$

The small angle between the tangent to the projected geodesic at P_1 and the straight line joining P_1 and P_2 is the *arc-to-chord* correction δ_{12} and is given by

$$\delta_{12} = -\frac{(N_2 - N_1)(E_2' + 2E_1')}{6r_m^2} \left\{ 1 - \frac{(E_2' + 2E_1')^2}{27r_m^2} \right\} \tag{7}$$

where $r_m^2 = \rho\nu k_0^2$ and ρ, ν are computed for $\phi_m = (\phi_1 + \phi_2)/2$. Equation (7) is given in various technical manuals (NMC 1972, NMC 1985 and ICSM 2002) and is regarded as accurate to about 0.02" over any 100 km line in a UTM zone. Bomford (1962) compared this formula with others over a known test line and recommended its use. For most practical purposes, the term in braces { } in equation (7) is omitted as its effect is negligible. For a line 100 km in length running north and south on a zone boundary the error in neglecting this term is about 0.08" (NMC 1985).

The arc-to-chord correction at P_2, for the line P_2 to P_1, is designated as δ_{21} and will be of opposite sign to δ_{12} and slightly different in magnitude. The arc-to-chord correction, grid bearing and plane bearing are related by

$$\theta = \beta + \delta \tag{8}$$

The grid convergence γ (given by Redfearn's equations) and the arc-to-chord corrections δ have a sign convention when used in Australia, given by the relationships in equations (5) and (8). Often the sign of these quantities can be ignored and the correct relationships determined from a simple diagram.

THE SURVEY AREA

The survey area of 5 kilometres by 5 kilometres is assumed to be near a UTM zone boundary, that is 3° from a central meridian of longitude λ_0. Victoria is covered by two UTM zones, 54 with $\lambda_0 = 141^\circ$ and 55 with $\lambda_0 = 147^\circ$ and the zone boundary between these zones is a meridian of longitude $\lambda = 144^\circ$. Also, Victoria is approximately contained within the parallels of latitude $\phi = -36^\circ$ and $\phi = -39^\circ$ (Wilson's Promontory is further south) and a mid-latitude for Victoria could be assumed to be $\phi = -37^\circ\,30'$. So, noting that 1° of latitude is approximately 111 kilometres and 5 kilometres very roughly equates to 2.5 minutes of arc, the centre of our survey area is assumed to be at $\phi_C = -37^\circ\,30'$ and $\lambda_C = 144^\circ\,02'\,30''$.

This would place it in zone 55, somewhere between Ballarat and Daylesford. Converting these geodetic coordinates to MGA coordinates (using **Redfearn.xls** with the parameters for the GRS80 ellipsoid) gives $E = 238482.350$ m, $N = 5845546.570$ m. Rounding these values to the nearest 500 metres and then adding and subtracting 2500 metres (2.5 km) gives the coordinates of our survey area as

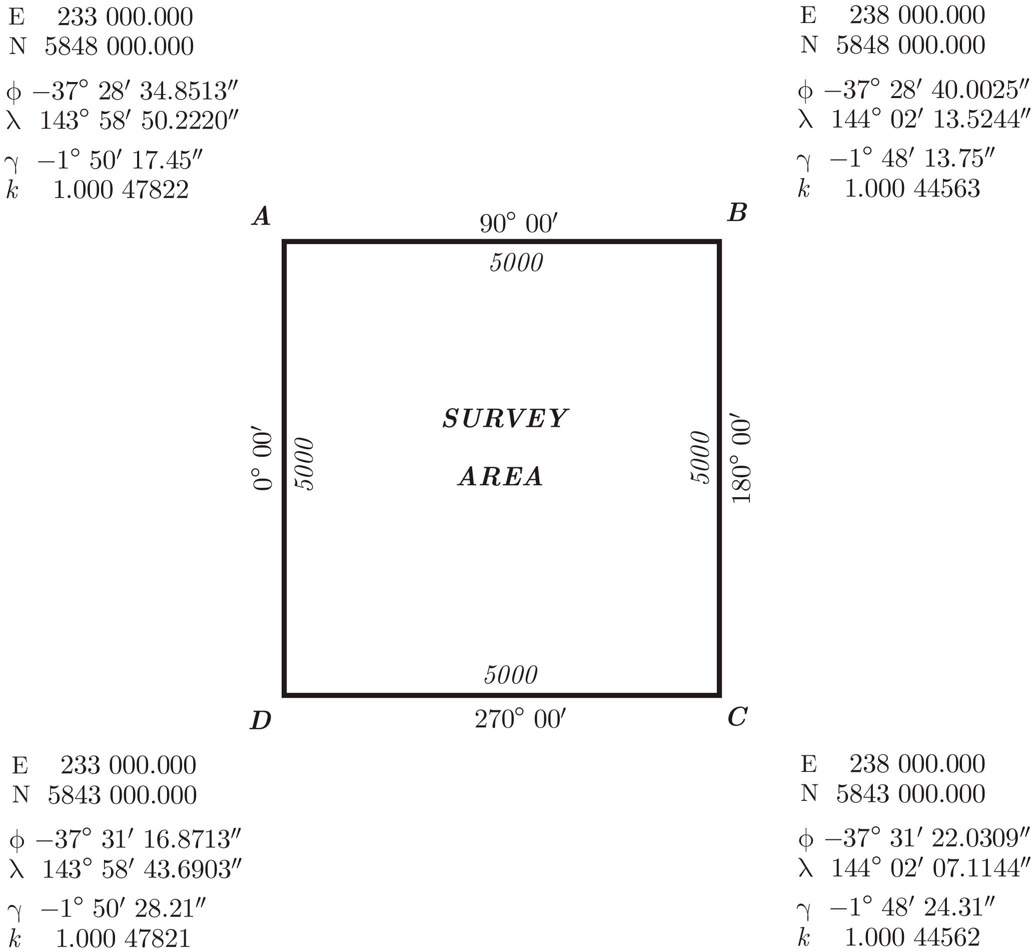


Figure 4 Coordinates of Survey Area

In Figure 4 the MGA grid coordinates are shown in each corner as well as latitudes ϕ, longitudes λ, grid convergences γ and point scale factors k. These latter values $(\phi, \lambda, \gamma, k)$ have been computed using **Redfearn.xls**.

ARC-TO-CHORD CORRECTIONS

Inspection of equation (7) reveals that maximum arc-to-chord corrections occur along north-south lines. Now, if we consider a 5 km line whose terminal points are A and D (the western edge of our survey area) with a mean latitude of $\phi_m = (\phi_A + \phi_D)/2 = -37^\circ\, 29'\, 55.8613''$ then $r_m = k_0\sqrt{\rho\nu}$ where $k_0 = 0.9996$ is the central meridian scale factor and ρ and ν are the radii of curvature of the meridian and prime vertical sections of the ellipsoid respectively. Using **Redfearn.xls** these values are (for ϕ_m) $\rho_m = 6359087.546$ m and $\nu_m = 6386063.012$ m and $r_m = 6370012$ m (to the nearest metre). The arc-to-chord correction of the line A–D using equation (7) is $\delta_{AD} = -3.39''$ and for the line D–A is $\delta_{DA} = 3.39''$ from this we may conclude that <u>for any line within our area there will be no difference between the magnitudes of the arc-to-chord corrections at either end of the line</u>. Also, the arc-to-chord corrections (ignoring the sign) for lines of 2.5 km and 1 km along the western boundary will be $\frac{1}{2}\delta_{AD} = 1.70''$ and $\frac{1}{5}\delta_{AD} = 0.68''$ respectively since the correction for north-south lines (the maximum correction) is proportional to the differences in northing coordinates.

We can also conclude that the arc-to-chord correction is insensitive to the value of r_m. For the same 5 km north-south line but with $\phi_m = -36^\circ$ and $r_m = 6368940$ m the correction is $\delta = 3.39''$ and for $\phi_m = -39^\circ$ and $r_m = 6371101$ m the correction is also $\delta = 3.39''$. From this we may conclude that a value of $r_m = 6370000$ m (the average of the two values above rounded to the nearest km) is suitable for computing arc-to-chord corrections for any line, anywhere in Victoria.

LINE SCALE FACTORS

Line Scale Factor $K = \dfrac{L}{s}$ is the ratio of plane distance L (the distance on the UTM projection) and the geodesic distance s (the shortest distance between two points on the ellipsoid) and K can be computed using equation (4). As we can see, K is a function of the east coordinates of the terminal points of a line and will be a maximum for north-south lines on a zone boundary (where E_1' and E_2' will be maximum). For the 5 km line A–D (the western edge of our survey area) with a mean latitude of $\phi_m = -37^\circ\, 29'\, 55.8613''$ and $r_m = 6370012$ m (to the nearest metre) the line scale factor $K = 1.0004782$. From Figure 4, where the point scale factors are shown, we can conclude that the line scale factor for the line A–D is the mean of the points scale factors at the terminal points. For the line A–B (the northern edge of our survey area) with a mean latitude of $\phi_m = -37^\circ\, 28'\, 37.4268''$ and $r_m = 6369996$ m (to the nearest metre) the line scale factor $K = 1.0004619$. From Figure 4, the mean of the point scale factors at A and B is 1.0004619; the same as the computed line scale factor K, and this relationship will also apply for the diagonal lines A–C and B–D. From this analysis, we may conclude that the line scale factor K is equal to the mean of the point scale factors at the terminal points of any line in Victoria not exceeding 5 km in length.

REDUCTION OF DISTANCES TO UTM PLANE DISTANCES USING THE COMBINED SCALE FACTOR

Before any MGA coordination can take place the local plane horizontal distances H measured in the field must be reduced to distances s on the ellipsoid and then, by the application of scale factors, to plane distances L on the UTM projection plane. In this paper, we are considering horizontal distances of 1 km or less and certain approximations will be made. These approximations may not be applicable to longer lines and The GDA Technical Manual has a detailed discussion on this topic (reduction of distances) as have several other authors, e.g., Featherstone & Rüeger (2000), Deakin (2005a). These sources of information should be consulted for longer lines.

Figure 5 shows P_1 and P_2 at ellipsoidal heights $h_1 = P_1Q_1$ and $h_2 = P_2Q_2$ above the ellipsoid measured along the normal at P_1 and P_2. The geoid is shown as a gently undulating surface (hugely exaggerated in this diagram) and N_1 and N_2 are geoid-ellipsoid separations at P_1 and P_2. N-values vary throughout Australia and can be computed from AUSGeoid98 by software available at Geoscience Australia's website (http://www.ga.gov.au/). For our survey area, somewhere between Ballarat and Daylesford, the N-value is approximately 5 metres. Instructions for computing N-values are given in a following section.

For most practical purposes, the relationship

$$h = \text{AHD height} + N \tag{9}$$

connects ellipsoidal heights, AHD heights and geoid-ellipsoid separations.

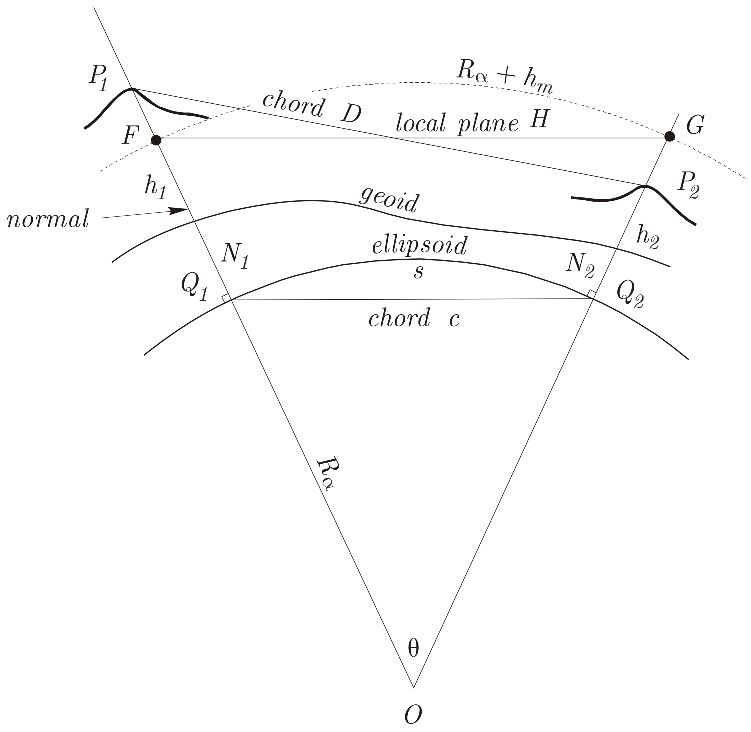


Figure 5. Geometry of reduction of local plane distances to the ellipsoid

In Figure 5, the ellipsoid is approximated by a circular arc of radius R_α where $R_\alpha = \dfrac{\rho_1 \nu_1}{\rho_1 \sin^2 \alpha_{12} + \nu_1 \cos^2 \alpha_{12}}$ is the radius of curvature at point 1 of the normal section in the direction α_{12} (α is azimuth and ρ, ν are radii of curvature in the meridian and prime vertical planes respectively). The chord distance $D = P_1 P_2$ is the measured slope distance (corrected for atmospheric conditions and instrumental errors) and FG is the chord distance D reduced for slope at a mean height $h_m = \dfrac{h_2 + h_1}{2}$ above the ellipsoid. [This is a reasonable assumption verified in Deakin (1984)]. We call the distance $H = FG$ the *Local Plane Distance* and assume that this is the horizontal distance displayed by the Total Station EDM and recorded in the field. s is the distance along the circular arc (approximating the ellipsoid) and c is the chord of the arc and s is regarded as being equivalent to the geodesic distance (the shortest path between two points on an ellipsoid).

By similar triangles in Figure 5, the chord c is given by

$$c = \left(\frac{R_\alpha}{R_\alpha + h_m} \right) H \tag{10}$$

The ratio $\dfrac{R_\alpha}{R_\alpha + h_m}$ can be regarded as a scale factor that we define as *Height Scale Factor*

$$\text{Height Scale Factor} = \frac{R_\alpha}{R_\alpha + h_m} \tag{11}$$

The chord c and the arc length s are connected by the relationship

$$c + corrn = s$$

where *corn* is a small unknown correction and from Figure 5

$$s = R_\alpha \theta \quad \text{and} \quad c = 2R_\alpha \sin\left(\frac{\theta}{2}\right) = 2R_\alpha \sin\left(\frac{s}{2R_\alpha}\right)$$

Using the series expansion for $\sin x = x - \dfrac{x^3}{3!} + \dfrac{x^5}{5!} - \cdots$ the difference ($corrn$) between the chord c and the arc s is given as

$$corrn = s - 2R_\alpha \left\{ \frac{s}{2R_\alpha} - \frac{1}{3!}\left(\frac{s}{2R_\alpha}\right)^3 + \cdots \right\} = \frac{s^3}{24R_\alpha^2} + \cdots$$

Using $R_\alpha = 6370000$ m (the average value for r_m used above) and $s = 5000$ m gives $corrn = 0.00012$ m. So, for our case, since we are only concerned with distance less than 1000 m (1 km) we can regard the chord c and the arc s as equal and using equation (10) the arc distance s is given by

$$s = \left(\frac{R_\alpha}{R_\alpha + h_m}\right) H \tag{12}$$

Now combining the definition of Line Scale Factor K [see equation (3)] with equations (11) and (12) gives an expression for the Plane Distance L as

$$L = Ks = K\left(\frac{R_\alpha}{R_\alpha + h_m}\right) H = \text{Line Scale Factor} \times \text{Height Scale Factor} \times H \tag{13}$$

The product of the two scale factors in equation (13) is known as the *Combined Scale Factor*

$$\text{Combined Scale Factor} = \text{Line Scale Factor} \times \text{Height Scale Factor} \tag{14}$$

and the Plane Distance L becomes

$$L = \text{Combined Scale Factor} \times H \tag{15}$$

ERRORS IN COMPUTATION OF *L* USING THE COMBINED SCALE FACTOR

Errors in the computation of Plane Distances L using the Combined Scale Factor can be investigated by using the Theorem of the Total Differential:

> If f is some function of independent variables x and y, i.e., $f = f(x,y)$ then $df = \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy$ where df, dx and dy are differentially small quantities and $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ are partial derivatives

Now the formula for computing the Plane Distance is $L = K\left(\frac{R_\alpha}{R_\alpha + h_m}\right)H$ and L is a function of the variables R_α and h_m or $L = L(R_\alpha, h_m)$ and $dL = \frac{\partial L}{\partial R_\alpha}dR_\alpha + \frac{\partial L}{\partial h_m}dh_m$. The differential dL can be thought of as a small error in the computed quantity L induced by small errors dR_α, dh_m in the variables R_α and h_m. The partial derivatives are $\frac{\partial L}{\partial R_\alpha} = \frac{h_m}{(R_\alpha + h_m)^2}(K \times H)$ and $\frac{\partial L}{\partial h_m} = \frac{-R_\alpha}{(R_\alpha + h_m)^2}(K \times H)$, and the error dL is given by

$$dL = \left\{\frac{h_m}{(R_\alpha + h_m)^2}dR_\alpha - \frac{R_\alpha}{(R_\alpha + h_m)^2}dh_m\right\}(K \times H) \qquad (16)$$

Now, let us assume that the mean height is $h_m = 500$ m and this value is known to be correct, i.e., $dh_m = 0$ and that the radius $R_\alpha = 6370000$ m is only approximate and could have an error $dR_\alpha = 20000$ m (20 km). What is the error dL in the computed distance L if $K = 1$ and $H = 1000$ m? Equation (16) gives $dL = 0.000246$ m. We may conclude from this that a value of $R_\alpha = 6370000$ m is suitable for calculating Height Scale Factor anywhere in Victoria.

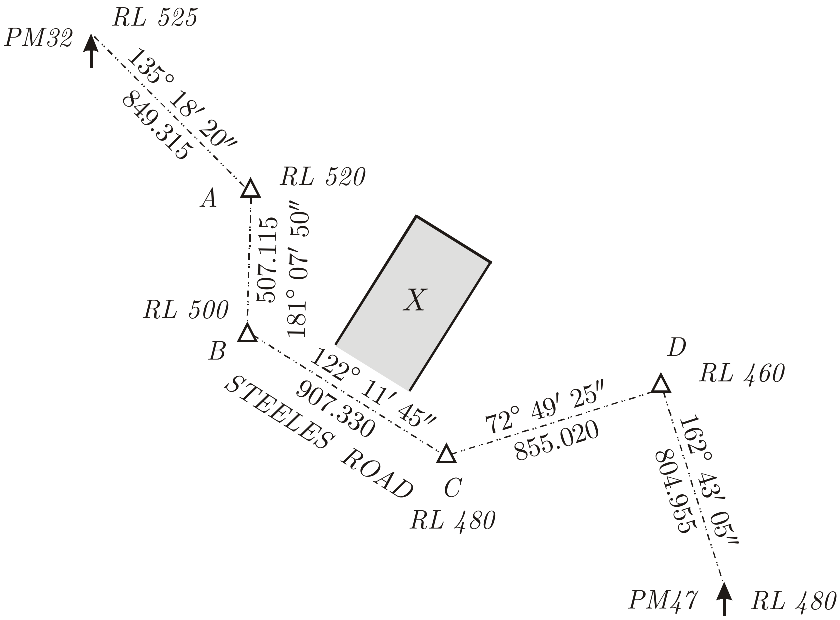
Now let us assume that $h_m = 500$ m but this value could be in error by 10 metres, i.e., $dh_m = 10$ m, and $R_\alpha = 6370000$ m with $dR_\alpha = 0$. What is the error dL in the computed distance L if $K = 1$ and $H = 1000$ m? Equation (16) gives $dL = -0.001570$ m, which is equivalent to 1.57 ppm for a 10 m error in the mean height.

Expressed another way, the errors in L are:

$$1 \text{ ppm for } dh_m \approx 6 \text{ m},$$
$$2 \text{ ppm for } dh_m \approx 12 \text{ m},$$
$$3 \text{ ppm for } dh_m \approx 18 \text{ m},$$
etc.

We may conclude from this that the mean ellipsoidal height h_m of any line needs to be known within ± 6 m to make sure that errors in the computation of L using the Combined Scale Factor do not exceed 1 ppm (0.001 m per kilometre). With this in mind, it is important that N-values are not neglected when determining the ellipsoidal heights h to be used in calculating Combined Scale Factors since $h = \text{AHD height} + N$ and the N-value could easily exceed 6 m. Furthermore, if it is decided to adopt an average height of an area for the purposes of computing a Combined Scale Factor for a survey area, then care should be taken in assessing the effects of undulating terrain.

AN EXAMPLE COMPUTATION



A survey is made of a rural property "X" on Steeles Road which is to be subdivided. For the purposes of coordination, the survey has been extended to two PM's that have known MGA coordinates. The bearing datum for the survey is related to original Crown Surveys and the distances shown are (local plane) horizontal distances. The Reduced Levels (RL's) at traverse points and the PM's are AHD but are only known to the nearest metre. The MGA Zone 55 coordinates are:

$PM32$	233 624.855 E	$PM47$	235 549.870 E
	5 848 077.325 N		5 845 514.270 N

STEP 1

Compute the bearing and distance between the PM's

PM32→PM47	Survey:	131° 11′ 36″	3204.245 m
	MGA:	143° 05′ 28″	3205.454 m

The difference in datum is 11° 53′ 52″ which is an approximate rotation from Survey to MGA plane bearings.

STEP 2

Apply the difference in datum and compute an approximate set of MGA coordinates of the traverse points. Nearest metre would be fine.

Line	Brg (approx MGA)	Dist	Point	E	N
PM32-A	147° 12′ 12″	849.315	A	234085	5847363
A-B	193° 01′ 42″	507.115	B	233971	5846869
B-C	134° 05′ 37″	907.330	C	234622	5846238
C-D	84° 43′ 17″	855.020	D	235474	5846317
D-PM47	174° 36′ 57″	804.955			

STEP 3

Compute Point Scale Factors k using approximate cords and **Redfearn.xls** (with parameters of GRS80 ellipsoid and MGA zone 55)

Point	E	N	PSF k
PM32	233624.855	5848077.325	1.0004741
A	234085	5847363	1.0004711
B	233971	5846869	1.0004718
C	234622	5846238	1.0004676
D	235474	5846317	1.0004620
PM47	235549.870	5845514.270	1.0004615

STEP 4

Compute Line Scale Factors K for each line, where K is the average of the Point Scale Factors at either end of the line.

Line	LSF K
PM32-A	1.0004726
A-B	1.0004714
B-C	1.0004697
C-D	1.0004648
D-PM47	1.0004618

STEP 5

(i) Compute the latitude and longitude of PM's 32 & 47 using **Redfearn.xls**

$$PM32 \quad \phi: \ -37^\circ\, 28'\, 32.9947'' \quad \lambda: 143^\circ\, 59'\, 15.7288''$$

$$PM47 \quad \phi: \ -37^\circ\, 29'\, 58.0403'' \quad \lambda: 144^\circ\, 00'\, 30.6846''$$

(ii) Compute the geoid-ellipsoid separation N at PM's 32 & 47 using the AUSGeoid software available at Geoscience Australia (http://www.ga.gov.au/) following the links to Geodesy & GPS then AUSGeoid then Compute an N value on-line (the input is ϕ, λ to the nearest second of arc)

$$PM32 \quad N: +5.546 \text{ m}$$
$$PM47 \quad N: +5.542 \text{ m}$$

Note that components of the deflection of the vertical are also computed but are not required.

STEP 6

Compute the mean ellipsoidal height h_m for each using an average N-value for the survey area, say 6 m (nearest metre), noting that $h = \text{AHD height} + N$. Then compute the Height Scale Factor using equation (11) with $R_\alpha = 6370000$ m and then the Combined Scale Factor using equation (14)

Line	*h* (mean)	HSF	LSF *K*	CSF
PM32-A	528	0.9999171	1.0004726	1.0003897
A-B	516	0.9999190	1.0004714	1.0003904
B-C	496	0.9999221	1.0004697	1.0003918
C-D	476	0.9999253	1.0004648	1.0003901
D-PM47	476	0.9999253	1.0004618	1.0003871

STEP 7

Compute the Plane Distances $L = \text{Combined Scale Factor} \times H$, where H is the traverse distance.

Line	Brg (approx MGA)	Plane Dist *L*
PM32-A	147° 12′ 12″	849.646
A-B	193° 01′ 42″	507.313
B-C	134° 05′ 37″	907.685
C-D	84° 43′ 17″	855.354
D-PM47	174° 36′ 57″	805.267

STEP 8

Recompute the bearing and distance between the PM's

PM32→PM47 Survey: 143° 05′ 28″ 3205.494 m

MGA: 143° 05′ 28″ 3205.454 m

COMMENTS ON THE COMPUTATION

1. No arc-to-chord corrections have been applied. Note that the largest arc-to-chord correction is $0.54''$ (line *D–PM47*)
2. The difference in bearings in Step 8 is zero so we may assume that the approximate MGA bearings shown in Step 7 are, for practical purposes, MGA Plane Bearings. MGA East and North coordinates can be computed for the traverse using the values in Step 7.
3. The difference in the distances *PM* to *PM* in Step 8 is 0.040 m, which is an accuracy of 1 in 80,000 (approximately) or 12 ppm. This may or may not be acceptable. If it was required to make the traverse "fit" the PM's, then an adjustment that changed only the distances could be used. (Crandall's adjustment is a least squares adjustment that changes only the distances)
4. A Combined Scale Factor for the rural property "X" could be adopted as 1.0003918 (the value for the line *C–D*).
5. This is not a real job. The data has been manufactured.

REFERENCES

Bomford, A.G., (1962). 'Transverse Mercator arc-to-chord and finite distance scale factor formulae', *Empire Survey Review,* No. 125, Vol. XVI, pp. 318-327, July 1962.

Deakin R.E., (1984). Computation of Spheroidal Distances, Lecture Notes, RMIT Department of Surveying, February 1984, 20 pages.

Deakin, R.E., (2005a). *Traverse Computation on the Ellipsoid and on the Universal Transverse Mercator projection,* School of Mathematical and Geospatial Sciences, RMIT University, Melbourne, 55 pages, March 2005.

Deakin, R.E., (2005b). 'Traverse computation: Ellipsoid versus the UTM projection', Presented at the Victorian/South Australia Survey Conference, *Water Wine Wind,* Mount Gambier, SA, 15-17 April 2005.

Featherstone, W.E. and Rüeger, J.M., (2000). 'The importance of using deviations of the vertical for reduction of survey data to a geocentric datum', *Trans tasman Surveyor,* Vol. 1, No. 3, pp. 46-61, December 2000, with Erratum: 'The importance of …', *The Australian Surveyor,* Vol. 47, No. 1, p. 7, June 2002.

Featherstone, W.E. and Kirby, J.F., (2002). 'Short note: Traverse computation on the ellipsoid instead of on the map plane', *The Australian Surveyor,* Vol. 47, No. 1, pp. 38-42, June 2002.

ICSM, (2002). *Geocentric Datum of Australia Technical Manual* – Version 2.2, Intergovernmental Committee on Surveying and Mapping (ICSM), February 2002, available online at: http://www.icsm.gov.au/icsm/gda/gdatm/index.html (last accessed March 2006)

NMC, (1972). *The Australian Map Grid Technical Manual,* Special Publication 7, National Mapping Council of Australia, Canberra, 1972

NMC, (1985). *The Australian Geodetic Datum Technical Manual,* Special Publication 10, National Mapping Council of Australia, Canberra, 1985

Redfearn, J.C.B., (1948). 'Transverse Mercator formula', *Empire Survey Review,* Vol. IX, No. 69, pp. 318-22.

Vincenty, T., (1975). 'Direct and inverse solutions of geodesics on the ellipsoid with application of nested equations', *Survey Review,* Vol. XXII, No. 176, pp. 88-93, April 1975.